Name ______________________

MATH 301 Differential Equations Fall 2005 Exam #1

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

You can use integration aids such as a table of integrals.

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1. Determine if the existence-uniqueness theorem guarantees a unique solution for the initial value problem (6 points)

$$\frac{dy}{dt} = |y|, \qquad y(0) = 1.$$

2. For each of the following, solve the given differential equation or initial-value problem. (15 points each)

   (a) $\dfrac{dy}{dt} = \dfrac{y^3 + 1}{ty^2}$ for $t > 0$

   (b) $\dfrac{dy}{dt} = \dfrac{y^2}{\sin y - 2ty}, \qquad y(1) = \pi/2$

   (c) $\dfrac{dy}{dt} = t^2 y^3, \qquad y(-1) = 2$

   (d) $\dfrac{dy}{dt} = \dfrac{3y^2 - t^2}{2ty}$ for $t > 0$ Hint: Use the substitution $v = \dfrac{y}{t}$.

3. Consider the differential equation $\dfrac{dy}{dt} = y(1 - y^2)$.

   (a) Sketch a slope field for with enough detail to show all interesting features. (8 points)

   (b) On your slope field, sketch enough solution curves to show all the possible types of behavior as t increases without bound. For each type of behavior, give the relevant range of initial values $y_0 = y(0)$ and briefly describe the behavior. (8 points)

4. A tank of volume V has an inlet pipe and an outlet pipe with equal flow rates f. The inlet pipe carries a chemical dissolved in water with a concentration that is decreasing exponentially. Model this concentration as $c_0 e^{-rt}$ for positive constants c_0 and r. For units, take t to be in minutes and c_0 to be in grams per liter.

   (a) Set up an initial value problem for the amount of chemical in the tank. Assume the tank is full of water and no chemical is in the tank at time $t = 0$. (7 points)

   (b) Solve the initial value problem you set up in (a). (8 points)

   (c) What is special about the case $r = \dfrac{f}{V}$? (3 points)